Quantum Simulation Of The Dirac Equation

[Introduction]

Original Authors of the Research Letter: R. Gerritsma, G. Kirchmair, F. Za¨hringer, E. Solano, R. Blatt & C. F. Roos

- The Dirac equation is famous for merging special relativity with quantum mechanics and predicting the existence of antimatter.
- It is also famous for its counterintuitive prediction of Zitterbewegung, Klein's Paradox and other quantum phenomenon.
- Zitterbewegung refers to the theoretical rapid fluctuation of a free particle caused by the interference of the positive and negative energy spinor states.
- The magnitude of the frequency and wavelength are beyond today's technological availability to be directly measured.
- We simulated Zitterbewegung using a controllable laboratory system that underlied the same mathematical model.
- The Dirac equation could be simulated using laser coupling to the three vibrational eigenmodes and the internal states of a single trapped ion.

Poster's Author Jiateng Gu (Nelson):

Email: zcapf41@live.ucl.ac.uk

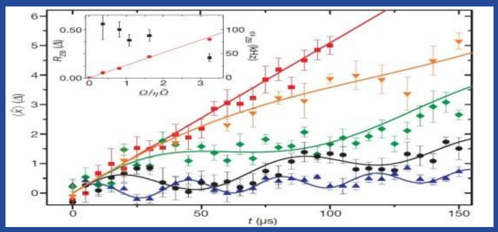
Degree: BSc Theoretical Physics University College London

[Theories & Techniques]

- The Dirac equation was simulated in 1+1 dimensions by trapping a single $^{40}Ca+$ ion in a linear Paul trap with laser coupling of axial trapping frequency 2π x 1.36 MHz and radial trapping frequency 2π x 3 MHz.
- Simplify the Dirac Hamiltonian H_D to $H_D = 2\eta\Delta\hat{\Omega}\sigma_x\hat{p} + \hbar\Omega\sigma_z$
- To study relativistic effects such as Zitterbewegung, it is necessary to measure $<\hat{x}(t)>$,the expectation value of the position operator of the harmonic oscillator. Note: $\hat{x}(t)$ describes the evolution of the particle
- To measure $<\hat{x}(t)>$for a motional state ρ_m, we have to (1) prepare the ion's internal state in an eigenstate of σ_y ,
- (2) Apply a unitary transformation, $U_{(\tau)}$, that maps information about ρ_m onto the internal states and
- (3) Record the changing excitation as a function of the probe time t, by measuring fluorescence.

[Data & Analysis]

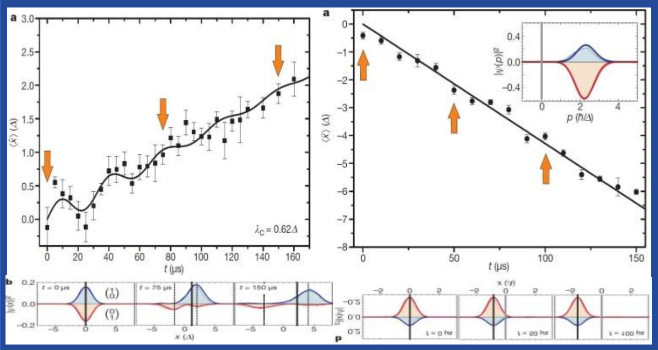
[Data 1]



This showed $<\hat{x}(t)>$ for particles with different masses

The linear curve (squares) represented a massless particle (Ω=0) moving at the speed of light. The solid curves represented numerical simulations. The figure showed Zitterbewegung for the crossover from the relativistic limit to the non-relativistic limit. It well fitted the theoretical prediction.

[Data 2]



Initially, Zitterbewegung appeared owing to interference of positive- and negative-energy parts of the state, $\psi(x;t=0) = e^{ix/\lambda_c}e^{-x^2/4\Delta^2}(\sqrt{2\pi}2\Delta)^{-1/2}\begin{pmatrix}1\\1\end{pmatrix}$ As these parts separated, the oscillatory motion faded away. Followed by the graph of the time evolution of a negative-energy eigenstate, it showed no Zitterbewegung.

Putting them together, it clearly showed that Zitterbewegung appeared only when there was interference of positive and negative energy parts of the state.

[Conclusion]

- It successfully simulated the Dirac equation under the simplified situation and showed approximately that long-argued Zitterbewegung to truly exist.
- It implemented a proof-of-principle quantum optical simulation of a tunable relativistic quantum mechanical system. It showed great potential and enormous possibilities that more complex quantum systems and theories may be well simulated in the near future.

[Reference]

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- Thaller, B. The Dirac Equation (Springer, 1992)